ON PERTURBATIONS ASSOCIATED WITH THE CREATION OF LIFT ACTING ON A BODY IN A TRANSONIC STREAM OF A DISSIPATIVE GAS

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The flow pattern of a viscous imcompressible fluid past a finite body is well known; an approximate solution of the related problem can, for example, be found in the book by Landau and Lifshits [1]. Finn [2] made a rigorous and exhaustive study of plane-parallel flows. No fundamental difficulties arise in passing from the motion of an incompressible fluid to a transonic flow of a compressible gas, however the velocity field is different, when the velocity of particles becomes critical at infinity.

The pattern of a sonic flow past a body of circular cross-section was investigated in paper [3]. This paper deals with perturbations associated with the creation of lift acting on an arbitrary body in a three-dimensional flow. When solving this problem it is necessary to consider not only the external stream, but also the laminar vortex trail because of the velocity vector transverse components becoming infinitely great, if functions defining these are formally extended into the trail area. This difficulty arises in investigations of three-dimensional flows only. The solution defining perturbation damping in an axisymmetric sonic stream of a dissipative gas has in its first approximation one singular point only, and does not contain any other singularities along the axis of symmetry [3].

The external stream pattern is essentially formed by the action of normal viscous stresses and the longitudinal component of the heat flux vector, while the distribution of gas parameters in the laminar trail is defined by tangential stresses. The conjunction of solutions valid for each of these areas makes the closure of the problem, and the determination of all necessary parameters possible.

1. The laminar trail. As the initial data we select the system of continuity equations, the Navier-Stokes and the heat transfer equations. Let x, y, and z denote the axes of a Cartesian coordinate system, v_x, v_y, and v_z velocity vector components along these axes, ρ the density, p the pressure, s the specific entropy, T the temperature, λ_1 the viscosity coefficient, λ_2 the secondary viscosity coefficient, and k the thermal conductivity. With these notations the Eqs. of gas motion are of the form [1]

$$\frac{\partial \rho v_x}{\partial x} + \frac{\partial \rho v_y}{\partial y} + \frac{\partial \rho v_z}{\partial z} = 0 \tag{1.1}$$

$$\rho \left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left[2\lambda_1 \frac{\partial v_x}{\partial x} + \left(\lambda_2 - \frac{2}{3} \lambda_1 \right) \times \right.$$
$$\left. \times \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) \right] + \frac{\partial}{\partial y} \left[\lambda_1 \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \right] + \frac{\partial}{\partial z} \left[\lambda_1 \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) \right] \tag{1.2}$$
$$(y, z)$$

$$\rho T\left(v_x \frac{\partial s}{\partial x} + v_y \frac{\partial s}{\partial y} + v_z \frac{\partial s}{\partial z}\right) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) +$$
$$+2\lambda_1\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2 + \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2 +\right.$$
$$\left. + \frac{1}{2}\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2\right] + \left(\lambda_2 - \frac{2}{3}\lambda_1\right)\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)^2 \qquad (1.3)$$

Here and in the following, symbol (y, z) indicates that equations which have been left out complete the projections of the full system onto the x, y, and z axes.

We close the above system with the following thermodynamic relationships [4]

$$ds = \frac{c_p}{\alpha\rho a^2 T}(dp - a^2 d\rho), \qquad dT = \frac{1}{\alpha\rho a^2}(\varkappa dp - a^2 d\rho)$$
$$\left(\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p, \quad a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s, \quad V = \frac{1}{\rho}, \quad \varkappa = \frac{c_p}{c_v}\right) \qquad (1.4)$$

Here $V = 1/\rho$ is the specific volume, α the coefficient of thermal expansion, a the adiabatic velocity of sound, c_p and c_v the specific heats at constant pressure and constant vólume respectively. Eqs. (1.4) make the elimination from our considerations of entropy and temperature possible.

In the following we shall consider the damping of perturbations at a considerable distance from the streamlined body. We subdivide the flow into two zones, one – the zone of the laminar vortex trail which extended downstream of the body in the form of a narrow tongue, and the second – the main (external) stream. These zones are denoted on Fig. 1 by the numerals *1* and *2* respectively. When considering small perturbations, we can assume that the values of gas parameters throughout the space under consideration differ but insignificantly from those in the stabilized and uniform free stream. We shall assume that the velocity of its particles coincides with the velocity of sound and is directed along the x-axis. Parameters of the medium in the unperturbed state will be denoted by an asterisk.

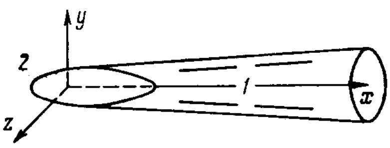

Fig. 1

In the trail downstream of the body the velocity field is essentially formed by the action of tangential viscous stresses. Phenomena in this area of space are close to those which occur in a boundary layer. Moreover, as was first noted by Tollmien [5], pressure variations may be disregarded when considering motions of an incompressible fluid, as such variations are negligible as compared with other gas-dynamic parameters. As the trail transverse dimensions are considerably smaller than the distance between the body and the selected cross-section, the derivatives taken along the y and z axes must considerably exceed the corresponding derivatives with respect to x.

We introduce into zone *1* the following dimensionless variables

$$x = \frac{l}{\Delta'}x', \qquad y = ly', \qquad z = lz', \qquad v_x = a_*(1 + \varepsilon' v_x') \qquad (1.5)$$
$$v_y = a_*\varepsilon' v_y', \qquad v_z = a_*\varepsilon' v_z', \qquad \rho = \rho_*(1 + \varepsilon'\rho'), \qquad p = p_*(1 + \varepsilon_p' p')$$

Here ε', ε_p' and Δ' are small numerical parameters, with $\varepsilon'_p \ll \varepsilon'$ by virtue of the assumed character of pressure variation. We select dimension l related to the trail cross-section as the characteristic unit of length. Formulas (1.5) do not hold for zone *2* of the external stream, where the correct estimate of the normal viscous stresses, and of the x-component of the heat flux has a decisive influence. As was first proved by Taylor [6], similar factors determine the pattern of weak shock waves.

Substituting Eqs. (1.5) into the system of Eqs. (1.1) to (1.3), we obtain three dimensionless coefficients, namely, two Reynolds and one Péclet numbers

$$N_{\mathrm{Re1}}' = \rho_* a_* l / \lambda_1, \qquad N_{\mathrm{Re2}}' = \rho_* a_* l / \lambda_2, \qquad N_{\mathrm{Pe}}' = \rho_* a_* c_p l / k \qquad (1.6)$$

computed from values of gas-dynamic functions applicable to the free sonic stream. We assume the reciprocal values of these parameters to be of the same order of magnitude, and are considerably smaller than unity. When deriving approximate equations we shall retain in these the dominant terms only, and neglect those of a higher order of smallness. Therefore, we can assume that in Eqs. (1.1) to (1.3) the coefficients of viscosity λ_1, λ_2 and of thermal conductivity k are constants, and equal to their respective values in the free stream. This remark also applies to the thermodynamic coefficients in Formulas (1.4).

As a result of linearization of the continuity equation we obtain

$$\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{1.7}$$

in which primes over all dimensionless variables have been omitted.

Eq. (1.7) shows that in any cross-section of the vortex trail a gas behaves as an incompressible fluid. A similar result is obtained in the case of the so-called 'slender body theory' for any sub-, tran-, and supersonic velocities of a stream in which viscous stresses and molecular heat transfer are absent [7].

In order to simplify the Navier-Stokes Eqs. (1.2) we may discard the nonlinear terms. A formal execution of this operation requires the existence of the following relationship between the small parameters ε', Δ' and $1/N'_{\mathrm{Re1}}$:

$$\varepsilon' \ll \Delta' = 1 / N'_{\mathrm{Re1}} \tag{1.8}$$

which we shall assume as satisfied. We finally obtain for the components of the velocity vector $\mathbf{V}$ the classical thermal conductivity Eq.

$$\frac{\partial v_x}{\partial x} = \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \qquad (y,z) \tag{1.9}$$

Turning to the heat influx Eq. (1.3) we find that it too yields an equation of heat conductivity

$$\frac{\partial \rho}{\partial x} = \frac{1}{N_{\mathrm{Pr}}'}\left(\frac{\partial^2 \rho}{\partial y^2} + \frac{\partial^2 \rho}{\partial z^2}\right) \tag{1.10}$$

which is satisfied by the perturbed gas density, and the coefficient of which is the Prandtl number $N_{\mathrm{Pr}}' = N_{\mathrm{Pe}}'/N_{\mathrm{Re1}}'$ computed with respect to the first Reynolds number. Thus in the approximation here considered the density variation is completely determined by thermal processes.

The system of three Eqs. (1.9) may be presented in the form of a single equation satisfying the perturbation velocity vector $\mathbf{V} = \mathbf{V}(v_x, v_y, v_z)$ of the gas particles. We shall look for a solution of the latter in the form

$$\mathbf{v} = \mathbf{u} + \operatorname{grad} \Phi \tag{1.11}$$

where the scalar function $\Phi(x, y, z)$ is defined by the thermal conductivity equation

$$\frac{\partial \Phi}{\partial x} = \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} \tag{1.12}$$

The results thus obtained will be subsequently applied to the computation of forces acting on the body. With this in view we subject function $\mathbf{u}(x, y, z)$ to initial values of the source type

$$\mathbf{u} = \mathbf{c}\delta(y)\delta(z) \qquad \text{for} \quad x = 0 \tag{1.13}$$

with vector $\mathbf{c} = \mathbf{c}(c_x, c_y, c_z)$ constant. Symbol $\delta(\alpha)$ denotes, as usually, the Dirac delta-function. The solution of the heat conductivity equation satisfying initial values (1.13) is well known, and can be expressed as [8]

$$\mathbf{u} = \frac{\mathbf{c}}{4\pi x}\left[\exp \frac{-(y^2 + z^2)}{4x}\right] \tag{1.14}$$

Scalar $\Phi(x, y, z)$ was introduced into representation (1.11) for the sake of satisfying the supplementary condition (1.7) imposed on the velocity vector transverse components. We have

$$\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = -\frac{\partial u_y}{\partial y} - \frac{\partial u_z}{\partial z} \tag{1.15}$$

The right-hand side of this equation must be equal to the derivative $\partial\Phi/\partial x$, as otherwise the thermal conductivity Eq. (1.12) would not have been satisfied. At first glance the problem so stated is insolvable, since one and the same function $\Phi(x, y, z)$ is defined as the solution of two different equations. This is, however not so. In fact, the differentiation of (1.15) with respect to x yields by virtue of Eqs. (1.9):

$$\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\frac{\partial\Phi}{\partial x} = -\frac{\partial}{\partial y}\frac{\partial u_y}{\partial x}-\frac{\partial}{\partial z}\frac{\partial u_z}{\partial x} = -\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)$$

Integrating this relationship we obtain

$$\frac{\partial\Phi}{\partial x} = -\frac{\partial u_y}{\partial y}-\frac{\partial u_z}{\partial z} \tag{1.16}$$

q.e.d.

Eq. (1.16) is the simplest for the purpose of determination of potential $\Phi(x, y, z)$, which is to remain constant along axis $y = z = 0$, and vanish when $y\to\infty$ and $z\to\infty$. Using Formula (1.14) for expressing derivatives $\partial u_y/\partial y$ and $\partial u_z/\partial z$, and satisfying stipulated conditions, we readily find

$$\Phi = \frac{c_y y + c_z z}{2\pi\,(y^2+z^2)}\left[\exp\frac{-(y^2+z^2)}{4x} - 1\right] \tag{1.17}$$

We change over to a cylindrical system of coordinates $y = r\cos\theta$, $z = r\sin\theta$. Velocity vector components v_r and v_θ are related to the Cartesian components v_y and v_z by Expressions $v_y = v_r\cos\theta - v_\theta\sin\theta,\quad v_z = v_r\sin\theta + v_\theta\cos\theta$

We now obtain as the result of combining Formulas (1.11), (1.14) and (1.17) the longitudinal component of the velocity vector

$$v_x = \frac{1}{4\pi x}\left[c_x + \frac{r\,(c_y\cos\theta + c_z\sin\theta)}{2x}\right]\exp\frac{-r^2}{4x} \tag{1.18}$$

The second term in brackets of (1.18) may be discarded, as its order of magnitude is considerably smaller than that of the first term, when the considered trail area is sufficiently far removed from the body. Similarly we find

$$\begin{aligned} v_r &= -\frac{c_y\cos\theta + c_z\sin\theta}{2\pi r^2}\left(\exp\frac{-r^2}{4x} - 1\right) \\ v_\theta &= \frac{c_z\cos\theta - c_y\sin\theta}{2\pi}\left[\frac{1}{2x}\exp\frac{-r^2}{4x} + \frac{1}{r^2}\left(\exp\frac{-r^2}{4x} - 1\right)\right] \end{aligned} \tag{1.19}$$

Here all terms are, however, of the same order of magnitude, and must be retained. With $r\to\infty$, i.e. at the trail external boundary, the transverse components v_r and v_θ of the velocity of particles tend to zero in accordance with the power law

$$v_r \to \frac{c_y\cos\theta + c_z\sin\theta}{2\pi r^2}, \qquad v_\theta \to \frac{c_y\sin\theta - c_z\cos\theta}{2\pi r^2} \tag{1.20}$$

while the damping of v_x proceeds exponentially, as shown by Eq. (1.18).

The trail density variation is found from Eq. (1.10). Its solution satisfying initial conditions $\rho = c_\rho\,\delta(y)\,\delta(z)$ for $x = 0$ may be expressed by

$$\rho = \frac{c_\rho N_{\mathrm{Pr}}'}{4\pi x}\exp\frac{-N_{\mathrm{Pr}}' r^2}{4x} \tag{1.21}$$

Solution (1.21) should only be used when friction and thermal processes in the body proximity substantially affect the entropy of particles flowing through the body surface boundary layer. The displacement of a finite mass of gas from the accompanying wake takes place then. A similar phenomenon occurs in flows past highly heated bodies transmitting considerable amounts of heat to the external flow. In conditions usual in aerodynamic problems concerning supersonic and transonic flows, the entropy of particles in the trail apparently differs but little from that of the free stream [9]. Hence, it can be assumed that $\rho = c_\rho = 0$. However at high supersonic velocities of flow past a body, the surface of the latter is heated to a high temperature.

2. The external flow. The motion of an incompressible fluid in zone *2* is found by solving the Laplace equation which is satisfied by the velocity potential [1]. At subsonic velocities a compressible stream behaves qualitatively as an incompressible fluid, however the velocity field undergoes a fundamental change, when the velocity of particles becomes critical at infinity. In that case it is no longer possible to consider the viscous stress tensor and the heat flux vector as being equal to zero. On the contrary, as was shown in [3], the gas motion in zone 2, at considerable distances from the body, is essentially determined by the action of normal viscous stresses and by heat transfer along the x-axis. This paper contains a generalization of the usual assumptions of the transonic flow theory; on the basis of these we write

$$x = Lx'', \qquad y = \frac{L}{\Delta''} y'', \qquad z = \frac{L}{\Delta''} z'', \qquad v_x = a_* (1 + \varepsilon'' v_x'') \tag{2.1}$$

$$v_y = \varepsilon'' \Delta'' a_* v_y'', \quad v_z = \varepsilon'' \Delta'' a_* v_z'', \quad \rho = \rho_* (1 + \varepsilon'' \rho''), \quad p = p_* (1 + \varepsilon'' p'')$$

where ε'' and Δ'' are numerical parameters of an order of magnitude considerably smaller than unity, and L is a characteristic length along the x-axis. Length L differs here from that selected for zone *1*, where it was related to a transverse dimension of the trail. As was shown by Taylor [6], formulas of the type of (2.1) define the pattern of weak shock waves.

The Reynolds and Péclet numbers reappear as coefficients in the initial Eqs. (1.1) to (1.3) when relationships (2.1) are substituted into these. As previously, we assume that the reciprocals of these numbers are of the same order of magnitude, and considerably smaller than unity. In the derivation of approximate equations we shall retain the dominant terms only. The derivation itself of the approximate equations will be analogous to that used in [3] for the analysis of an axisymmetric stream. Hence final results only will be adduced.

The linearization of the continuity equation and the projection of the Navier-Stokes Eq. (1.2) onto the x-axis yield

$$\rho = \frac{p_*}{\rho_* a_*^2} p = -v_x \tag{2.2}$$

where, as previously, primes over dimensionless parameters have been omitted.

Taking into account Formulas (2.2) we obtain the following relationships by projecting the Navier-Stokes equation onto the y and z axes

$$\frac{\partial v_x}{\partial y} = \frac{\partial v_y}{\partial x}, \qquad \frac{\partial v_x}{\partial z} = \frac{\partial v_z}{\partial x} \tag{2.3}$$

Noting that equality

$$\frac{\partial v_y}{\partial z} = \frac{\partial v_z}{\partial y}$$

is valid for $x \to -\infty$, we can derive from Eqs. (2.3) the complete condition for an unperturbed vortex-free external flow, i.e. rot $\mathbf{v} = 0$, hence $\mathbf{v} = \operatorname{grad} \varphi$.

The heat influx Eq. (1.3) requires a preliminary transformation by combining it with the continuity and the Navier-Stokes equations. Further to this it is essential for the derivation of the final relationship to establish the interdependence of the small parameters ε'', Δ'' and $1/N_{\mathrm{Re}}''$ which was derived in [3]

$$\varepsilon'' \ll \Delta''^2 = \frac{1}{N_{\mathrm{Re}}''}\left(1 + \frac{\varkappa - 1}{N_{\mathrm{Pr}}''}\right) \qquad \left(\frac{1}{N_{\mathrm{Re}}''} = \frac{4}{3}\frac{1}{N_{\mathrm{Re1}}''} + \frac{1}{N_{\mathrm{Re2}}''},\ N_{\mathrm{Pr}}'' = \frac{N_{\mathrm{Pe}}''}{N_{\mathrm{Re}}''}\right)$$

Here the Prandtl number is computed with respect to the combined Reynolds number based on the so-called 'longitudinal viscosity', and not to the first one.

As a result we have

$$\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{2.4}$$

The Cartesian coordinates used so far are convenient for qualitative estimates and derivation of asymptotic equations. For zone 2, however, a simpler form of solution is obtained in cylindrical coordinates. For components v_r and v_θ of the velocity of particles we have instead of (2.3)

$$\frac{\partial v_x}{\partial r} = \frac{\partial v_r}{\partial x}, \qquad \frac{1}{r}\frac{\partial v_x}{\partial \theta} = \frac{\partial v_\theta}{\partial x} \tag{2.5}$$

and Eq. (2.4) becomes

$$\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} = 0 \tag{2.6}$$

Eqs. (2.5) and (2.6) constitutes a closed system invariant with respect to the continuous two-parameter group of similarity transformations

$$x \to \alpha x, \quad r \to \alpha^{3/2} r, \quad \theta \to \theta$$

$$v_x \to \alpha^{-3n/2} v_x, \quad v_r \to \alpha^{-(3n+1)/2} v_r, \quad v_\theta \to \alpha^{-(3n+1)/2} v_\theta$$

Hence the existence for this system of self-similar solutions of the form (2.7)

$$v_x = r^{-n} w_x(\xi, \theta), \quad v_r = r^{-(3n+1)/3} w_r(\xi, \theta), \quad v_\theta = r^{-(3n+1)/3} w_\theta(\xi, \theta), \quad \xi = x r^{-2/3}$$

A solution of this kind yields for $n = 4/3$ and $v_\theta = \partial/\partial\theta = 0$ the asymptotic laws of perturbation damping in zone 2 at considerable distances from a body of revolution in a stream of viscous and heat conducting gas the velocity of which becomes sonic at infinity [3]. Formula (2.7) will obviously also define for $n = 4/3$ components of the particles velocity vector which are associated with the drag acting on a body of an arbitrary form. We shall find the magnitude of the divergence of gas parameters from their equilibrium values resulting from the presence of a lift acting on a body in a stream at critical velocity.

For the formulation of the solution we shall use, as previously, Formulas (2.7) the substitution of which into the system of Eqs. (2.5) and (2.6) yields

$$\frac{2}{3}\xi\frac{\partial w_x}{\partial \xi} + \frac{\partial w_r}{\partial \xi} + n w_x = 0, \qquad \frac{\partial w_x}{\partial \theta} - \frac{\partial w_\theta}{\partial \xi} = 0$$

$$\frac{\partial^2 w_x}{\partial \xi^2} - \frac{2}{3}\xi\frac{\partial w_r}{\partial \xi} + \frac{\partial w_\theta}{\partial \theta} - \left(n - \frac{2}{3}\right) w_r = 0$$

The exponent is not known a priori, it is however obvious that $n > 4/3$.

These equations contain functions of two independent variables ξ and θ. In order to further simplify the looked for solution we expand it into a Fourier series in which the first harmonic terms only will be retained. This procedure is in full accord with Formulas (1.18) and (1.19) which, in fact, also represent first terms of a Fourier expansion of the solution for the accompanying trail zone. We thus have

$$w_x = f(\xi)\,(c_1 \sin\theta + c_2 \cos\theta), \qquad w_r = g(\xi)\,(c_1 \sin\theta + c_2\cos\theta)$$

$$w_\theta = h(\xi)\,(c_2 \sin\theta - c_1 \cos\theta) \tag{2.8}$$

with arbitrary constants c_1 and c_2. The following system of equations is valid for functions $f(\xi)$, $g(\xi)$ and $h(\xi)$:

$$\frac{2}{3}\xi\frac{df}{d\xi} + \frac{dg}{d\xi} = -nf, \quad \frac{dh}{d\xi} = -f, \quad \frac{d^2 f}{d\xi^2} = \frac{2}{3}\xi\frac{dg}{d\xi} + \left(n - \frac{2}{3}\right) g - h$$

This system is equivalent to the single Eq.

$$\frac{d^3 f}{d\xi^3} + \frac{4}{9}\xi^2\frac{d^2 f}{d\xi^2} + \frac{4}{3}\xi\left(n + \frac{1}{3}\right)\frac{df}{d\xi} + (n^2 - 1) f = 0 \tag{2.9}$$

Having solved Eq. (2.9) we find functions $g(\xi)$ and $h(\xi)$ from the following equalities:

$$g = \frac{3}{3n-2}\left(\frac{d^2 f}{d\xi^2} + \frac{4}{9}\xi^2\frac{df}{d\xi} + \frac{2}{3}n\xi f + h\right), \qquad h = -\int_{-\infty}^{\xi} f(\xi)\, d(\xi) \tag{2.10}$$

The absolute value of the self-similar coordinate increases infinitely when $r \to 0$. We shall write down the asymptotic expansions for the three linearly independent solutions of Eqs. (2.9) for $|\xi| \to \infty$. The first of these is

$$f = a_1 |\xi|^{-3(n+1)/2}\,[1 + {}^{27}/_{64}(n+1)(n + {}^5/_3)(n + {}^7/_3)\,\xi^{-3} + \cdots] \tag{2.11}$$

The second of the looked for solutions will be presented in the form

$$f = a_2 |\xi|^{-3(n-1)/2}\,[1 - {}^{81}/_{32}(n-1)(n - {}^1/_3)(n + {}^1/_3)\,\xi^{-3}\ln|\xi| + \cdots] \tag{2.12}$$

and the third linearly independent solution of Eq. (2.9) will be written thus

$$f = a_3 |\xi|^{3(n-1)} \exp(-{}^4/_{27}\xi^3) + \cdots \qquad (2.13)$$

It will be readily seen from equalities (2.7), (2.8) and (2.10) that for $r \to 0$ and $x < 0$ the first terms of the asymtotic expansion of functions $v_x(x, r, \theta)$, $v_r(x, r, \theta)$ and $v_\theta(x, r, \theta)$ corresponding to Formula (2.11) are

$$v_x = a_1 |x|^{-3(n+1)/2} r (c_1 \sin\theta + c_2 \cos\theta) + \cdots$$

$$v_r = -\frac{2a_1}{3n+1} |x|^{-(3n+1)/2} (c_1 \sin\theta + c_2 \cos\theta) + \cdots$$

$$v_\theta = \frac{2a_1}{3n+1} |x|^{-3(n+1)/2} (c_2 \sin\theta - c_1 \cos\theta) + \cdots$$

The formula for $v_x(x, r, \theta)$ remains unchanged for $r \to 0$ and $x > 0$, while expressions of $v_r(x, r, \theta)$ and $v_\theta(x, r, \theta)$ change because of the presence of integral terms in the right-hand sides of equalities (2.10). We have

$$v_r = -\frac{3B_1}{3n-2} r^{-(3n+1)/3} (c_1 \sin\theta + c_2 \cos\theta) + \cdots \qquad (2.14)$$

$$v_\theta = -B_1 r^{-(3n+1)/3} (c_2 \sin\theta - c_1 \cos\theta) + \cdots \quad \left(B_1 = + \int_{-\infty}^{+\infty} f(\xi)\, d\xi\right)$$

The longitudinal component of the perturbed stream velocity computed from solution (2.12) tends to increase infinitely all along the x-axis as $1/r$, while its transverse components increase as $1/r^2$ when $x < 0$, and as $r^{-(3n+1)/3}$ when $x > 0$, with Formulas (2.14) remaining valid for these. As regards solution (2.13) we note that it yields expressions containing the common factor $\exp(-4x^3/27r^2)$, and is valid for all three components of the gas particles velocity. Therefore solution (2.13) does not hold for negative values of x, while, on the other hand, perturbations corresponding to positive values of x are extremely rapidly attenuated when $r \to 0$.

Upstream, the perturbations defined by integrals (2.9) must fade out at infinity. The area downstream of the body is occupied by a vortex trail, hence the two transverse components v_r and v_θ of the external stream velocity vector will have singularities defined by Formulas (1.20) when $r \to 0$ and $x > 0$. The perturbed velocity longitudinal component has no singularities, as at the trail boundary it becomes, in accordance with equality (1.18), exponentially small. We note that in the solution of an axisymmetric problem of perturbations created by drag, the gas motion outside of the trail is determined independently of the motion within the latter [3]. The solution of a three-dimensional problem is considerably more complicated.

By combining the functions which define components v_r and v_θ in-, and outside of the vortex trail in the interval $1 \ll \xi \ll r^{4/3} (\Delta' \Delta''^{2/3} lL^{1/3})^{-1}$ (coordinate r is here dimensional), we obtain first of all $(3n + 1)/3 = 2$, hence $n = 5/3$. With this value of exponent n a single integration of Eq. (2.9) becomes possible

$$\frac{d^2 f}{d\xi^2} + \frac{4}{9}\xi^2 \frac{df}{d\xi} + \frac{16}{9}\xi f = 0 \qquad (2.15)$$

In order to satisfy the natural condition for the damping of perturbations at infinity upstream of the body, the arbitrary constant in the above equation has been assumed to be zero. The asymptotic expansion of the first linearly independent solution of the above equation for $|\xi| \to \infty$ is provided by Formula (2.11), and for the second by (2.13). In order to find the explicit expression of function $f(\xi)$ we introduce into Eq. (2.15) a new independent variable $\eta = -4/27\,\xi^3$, and obtain

$$\eta \frac{d^2 f}{d\eta^2} + ({}^2/_3 - \eta) \frac{df}{d\eta} - {}^4/_3 f = 0$$

The general solution of the derived equation, presented in the standard form of confluent hypergeometric functions, is [10]

$$f = b_1 \Phi({}^4/_3, {}^2/_3; \eta) + b_2 \eta^{1/3} \Phi({}^5/_3, {}^4/_3; \eta) \qquad (2.16)$$

There remains to be established the relationship between constants b_1 and b_2. To achieve this we shall use the asymptotic presentation of confluent hypergeometric functions for $\eta \to +\infty$. We have

$$f = \eta^{2/3} e^{\eta} G(-{}^2/_3, -{}^1/_3; \eta)\left[\frac{\Gamma({}^2/_3)}{\Gamma({}^4/_3)} b_1 + \frac{\Gamma({}^4/_3)}{\Gamma({}^5/_3)} b_2\right] + \ldots$$

where $\Gamma(\alpha)$ is the Euler gamma-function, and $G(-{}^2/_3, -{}^1/_3; \eta)$ a series expansion in reciprocals of powers of η, with $G(-{}^2/_3, -{}^1/_3; \eta) \to 1$ when $\eta \to +\infty$. To obtain a solution guaranteeing the damping of perturbations in the external stream at infinity upstream of the body, it is necessary to set

$$\frac{b_2}{b_1} = -\frac{6\Gamma^2({}^2/_3)}{\Gamma^2({}^1/_3)}$$

Formula (2.16) is now transformed into

$$f = b_1\left[\Phi\left(\frac{4}{3}, \frac{2}{3}; \eta\right) - \frac{6\Gamma^2({}^2/_3)}{\Gamma^2({}^1/_3)} \eta^{1/3} \Phi\left(\frac{5}{3}, \frac{4}{3}; \eta\right)\right] \tag{2.17}$$

whence passing to the Ψ-function, first used in problems of mathematical physics by Tricomi [10], we find

$$f = \frac{2\Gamma({}^2/_3)}{3\Gamma({}^1/_3)} b_1 \Psi\left(\frac{4}{3}, \frac{2}{3}; \eta\right) \tag{2.18}$$

Equalities (2.10) permit the expressions of $g(\xi)$ and $h(\xi)$ to be written in the form

$$g = -\frac{4\Gamma({}^2/_3)}{9\Gamma({}^1/_3)} b_1 \xi \Psi\left(\frac{4}{3}, \frac{2}{3}; \eta\right) - \frac{2\Gamma({}^2/_3)}{3\Gamma({}^1/_3)} b_1 \int_{-\infty}^{\xi} \Psi\left(\frac{4}{3}, \frac{2}{3}; \eta\right) d\xi$$

$$h = -\frac{2\Gamma({}^2/_3)}{3\Gamma({}^1/_3)} b_1 \int_{-\infty}^{\xi} \Psi\left(\frac{4}{3}, \frac{2}{3}; \eta\right) d\xi$$

Using the asymptotic expansion of the Ψ-function for large values of the argument, we obtain for $\eta \to +\infty$ and $\xi \to -\infty$ the following form of solution (2.18)

$$f = \frac{2\Gamma({}^2/_3)}{3\Gamma({}^1/_3)} b_1 \eta^{-4/3} + \cdots = \frac{27 \cdot 2^{1/3}\Gamma({}^2/_3)}{4\Gamma({}^1/_3)} b_1 \xi^{-4} + \cdots \tag{2.19}$$

and the formula defining the behavior of functions $g(\xi)$ and $h(\xi)$

$$g = -h = -\frac{9 \cdot 2^{1/3}\Gamma({}^2/_3)}{4\Gamma({}^1/_3)} b_1 \xi^{-3} + \cdots \tag{2.20}$$

The original presentation of function $f(\xi)$ together with the asymptotic expressions of confluent hypergeometric functions in the area of negative changes of the argument, is more convenient for the analysis of that function for $\eta \to -\infty$ and $\xi \to +\infty$

$$f = -\frac{4\Gamma({}^2/_3)}{3\Gamma({}^1/_3)} b_1 \eta^{-4/3} + \ldots = -\frac{27 \cdot 2^{1/3}\Gamma({}^2/_3)}{2\Gamma({}^1/_3)} b_1 \xi^{-4} + \ldots \tag{2.21}$$

Functions $g(\xi)$ and $h(\xi)$ obviously tend to the constant value

$$-B_1 = -\int_{-\infty}^{+\infty} f(\xi)\, d\xi = \frac{2\Gamma({}^2/_3)}{3\Gamma({}^1/_3)} b_1 \int_{-\infty}^{+\infty} \psi\left(\frac{4}{3}, \frac{2}{3}; \eta\right) d\xi \tag{2.22}$$

when $\eta \to -\infty$ and $\xi \to +\infty$.

Functions $f(\xi)$, $g(\xi)$ and $h(\xi)$ computed for $b_1 = 4\Gamma(1/3)[27 \cdot 2^{1/3}\Gamma(2/3)]^{-1}$, with constant B_1 obtained equal to 0.667, are shown on Fig. 2. In accordance with Formulas (2.19) to (2.22) functions $f(\xi)$ and $g(\xi)$ pass through zero once, and have different signs for $\xi \to -\infty$ and $\xi \to +\infty$. The sign of function $h(\xi)$ remains unchanged and is negative throughout the interval of change of ξ.

We shall carry out the complete conjugation of solutions related to the vortex trail and to the flow outside of its boundary. Components v_r and v_θ of the basic stream velocity vec-

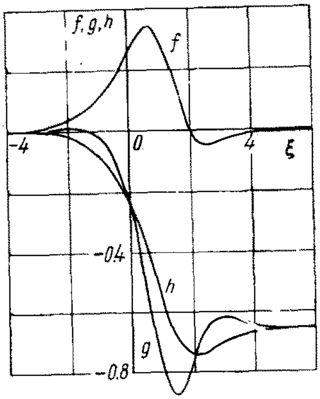

Fig. 2

tor must have, as previously indicated, singularities of the form of (1.20). Recalling the manner in which the dimensionless coordinate r was introduced into zones *1* and *2*, we have by virtue of equalities (2.7), (2.8) and (2.22)

$$c_1 = -\frac{\Delta''^2}{2\pi B_1}\left(\frac{l}{L}\right)^2 C_z, \qquad c_2 = -\frac{\Delta''^2}{2\pi B_1}\left(\frac{l}{L}\right)^2 C_y \tag{2.23}$$

We note that it is not possible to "match" the solution valid for the trail and that applicable outside of it for any arbitrary values of $n \neq 5/3$, because of the four contradictory equalities relating constants c_1 and c_2 to constants C_y and c_z. These two pairs of equalities are congruent for $n = 5/3$ only.

If the gas motion in the trail downstream of the body is axisymmetric, then, in accordance with (1.19), we have $v_r = v_\theta = c_y = c_z = 0$. In this case the order of magnitude of the transverse components of the velocity vector is considerably smaller than that of the longitudinal one, not only in zone *2*, but also in zone *1*. Relationship (1.7) no longer holds for the simplified continuity equation, and terms containing derivatives with respect to x must be retained in it. Exactly the same situation occurs in the analysis of plane parallel flows. Thus it is clear that in the case of a flow past a body of circular cross-section under zero angle of attack, the external stream in the approximation here considered is independent of the velocity field pattern in the accompanying trail.

3. Forces acting on a body. Denoting temporarily for the sake of convenience the Cartesian coordinates by $x = x_1$, $y = x_2$ and $z = x_3$, and using the customary notations for sums with recurring indices, we adduce the expressions of components π_{ij} of the density tensor of momentum flow [1]

$$\pi_{ij} = p\delta_{ij} + \rho v_i v_j - \lambda_1\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial v_k}{\partial x_k}\right) - \lambda_2\delta_{ij}\frac{\partial v_k}{\partial x_k} \tag{3.1}$$

Here all parameters are expressed in dimensional units of the input system, and δ_{ij} is the unit vector, i.e. $\delta_{ij} = 1$ for $i = j$, and $\delta_{ij} = 0$ for $i \neq j$, with indices i, j, k run through values 1, 2, 3.

Let us surround the stream flowing past a body by a closed surface Σ. The total force **F** acting on the body is equal to the integral of the density tensor of the momentum flow taken over this surface. The components of this force are [1]

$$F_i = -\oint_\Sigma \pi_{ij} d\sigma_j \tag{3.2}$$

The absolute value of vector $d\sigma$ is equal to the surface element area, and is directed outward along a normal to the latter. When computing integral (3.2) note must be taken of

$$\oint_\Sigma \rho v_j d\sigma_j = 0 \tag{3.3}$$

as the quantity of gas in the volume under consideration remains constant. Condition (3.3) is the integral form of the continuity equation.

We select for our investigations the volume of gas bounded by two planes $x = \text{const}$, one of which denoted by Σ_1, is located sufficiently far upstream from the body, and the other, denoted by Σ_2, downstream of it. Let a cylindrical surface Σ_0 of radius R, symmetric with respect to the x-axis, envelop the space comprised by these two planes; this radius will be subsequently extended to infinity. The area of the vortex trail cross-section at plane Σ_2 will be denoted by S.

The primary object of this paper is the computation of the lift and lateral force denoted respectively by F_y and F_z. In addition to this we shall derive the expression for the drag F_x, although the laws of damping of perturbations generated by it are already known.(*) We

*) The derivation of the formula for F_x in [3] is erroneous.

proceed with the determination of these by dividing integral (3.2) into two terms, one of which denoted by F_i' represents that part of forces which is obtained by the integration of tensor π_{ij} over the trail area, and the other denoted by F_i'' contributed by the integration of functions defining the external stream. Obviously, $F_i = F_i' + F_i''$.

In the computation of F_i' in Formula (3.1) it will be sufficient to take into account only the terms related to the mechanical displacement of the various gas masses from one place to another, together with pressure forces acting there. We substitute into the right-hand side of equality (3.2) the dimensionless variables (1.5), retaining the primes over these in order to avoid any subsequent confusion. After some simple computations we find the magnitude

$$F_x' = -\rho_* a_* l^2 \varepsilon' \iint\limits_S v_x' dy' dz' \tag{3.4}$$

which is a component of drag. Noting that function $v_x'(x', r')$ decreases exponentially when $r' \to \infty$, we can, in the approximation considered here, extend the integration of (3.4) over the whole plane Σ_2. With the use of Formula (1.18) we obtain

$$F_x' = -\rho_* a_*^2 l^2 c_x \varepsilon' \tag{3.5}$$

For components F_y' and F_z' of the lift and lateral force respectively the following relationships are true

$$F_y' = -\rho_* a_*^2 l^2 \varepsilon' \iint\limits_S v_y' dy' dz', \qquad F_z' = -\rho_* a_*^2 l^2 \varepsilon' \iint\limits_S v_z' dy' dz'$$

Reverting to equalities (1.19) we note that the integration of terms dependent on gradΦ as defined by (1.11) for the perturbed velocity, yields zero. Only terms containing $\mathbf{u}(x', y', z')$ in (1.11) yield finite contributions to F_y' and F_z'. Substituting integration over Σ_z for integration over S, and neglecting the ensuing error, we obtain

$$F_y' = -\rho_* a_*^2 l^2 c_y \varepsilon', \qquad F_z' = -\rho_* a_*^2 l^2 c_z \varepsilon' \tag{3.6}$$

By virtue of equality (2.23) constant c_1 is proportional to constant c_z, and constant c_2 to c_y. All of these constants vanish in the absence of lift and lateral forces, and in that case both, the motion of gas in the vortex trail and the flow in the area outside of the latter are axisymmetric. The axisymmetric part of perturbations is due to drag only. When integrating external flow functions over the two end planes $x = \text{const}$ for the purpose of drag calculation, it is necessary to take into account in Formula (3.1) not only the "ideal" part, but also the terms contributed by viscous friction. For the determination of this force the first approximation theory is not sufficient, and it is necessary to know the corrections derived in the second approximation theory. Its simplifaction is based on the previously noted fact that gas parameters in zone 2 are found by solving the axisymmetric problem, and depend on the x, and r coordinates only, while $\partial_\theta = \partial / \partial\theta = 0$.

According to the theory developed in [12] perturbations are presented in the form

$$\begin{aligned} v_x'' &= v_{x1}'' + \delta'' v_{x2}'' + \cdots, & v_r'' &= v_{r1}'' + \delta'' v_{r2}'' + \cdots \\ \rho'' &= \rho_1'' + \delta'' \rho_2'' + \cdots, & p'' &= p_1'' + \delta'' p_2'' + \cdots \end{aligned} \tag{3.7}$$

where δ'' is a small supplementary parameter, and functions $\rho_2''(x'', r'')$ and $p_2''(x'', r'')$ are given in terms of $v_{x1}''(x'', r'')$ and $v_{x2}''(x'', r'')$ by equalities

$$\rho'' = A^{-1}\left(1 + \frac{\varkappa - 1}{N_{\mathrm{Pr}}''}\right)^2 \frac{\partial v_{x1}''}{\partial x''} - v_{x2}'', \qquad p_2'' = \frac{\rho_* a_*^2}{p_*}\left[A^{-1}\left(1 + \frac{\varkappa - 1}{N_{\mathrm{Pr}}''}\right)\frac{\partial v_{x1}''}{\partial x''} - v_{x2}''\right]$$

$$A = \frac{\varkappa}{N_{\mathrm{Pr}}''} + \left(1 + \frac{\varkappa - 2}{N_{\mathrm{Pr}}''}\right)\left(1 + \frac{\varkappa - 1}{N_{\mathrm{Pr}}''}\right) \tag{3.8}$$

Function $v_{x1}''(x'', r'')$ is, of course, different from that considered in the preceding Section, where it is related to forces acting in the yz-plane. This function was defined in [3]. A more precise definition of gas characteristics in the area of perturbed motion makes it possible to establish the relationship between the small parameters [11]

$$\Delta'' = \varepsilon''^{1/4} (2m_*)^{1/4} A^{-1/4} \left(1 + \frac{\varkappa - 1}{N_{\mathrm{Pr}}''}\right)^{1/2}, \qquad \delta'' = \varepsilon''^{1/2} (2m_*)^{1/2} A^{1/2} \left(1 + \frac{\varkappa - 1}{N_{\mathrm{Pr}}''}\right)^{-1}$$

$$\frac{1}{N_{\text{Re}}''} = \varepsilon''^{1/2}(2m_*)^{1/2}A^{-1/2}, \qquad m_* = \frac{1}{2\rho_*{}^3a_*{}^2}\left(\frac{\partial^2 p}{\partial V_*{}^2}\right)_s \tag{3.9}$$

In spite of the use of the second approximation theory, the contribution of nonlinear terms to the value of tensor π_{ij} may be neglected.

We shall substitute dimensionless variables (2.1) into the right-hand side of equality (3.2). First of all, we shall demonstrate with the use of Formulas (3.7) to (3.9) that the term F_x'' vanishes in the expression of drag. It can be readily ascertained that the integral in the expression of F_x'' taken over the cylindrical surface Σ_0 tends to vanish when its radius $R \to \infty$. We note at once that the integrals over Σ_0 appearing in the formulas for the lift and lateral force also tend to vanish when $R \to \infty$. In the summation in π_{xx} of terms associated with pressure and velocity perturbations the coefficient in front of function $v_{x2}''(x'', r'')$ of the second approximation becomes zero, hence

$$F_x'' = \rho_* a_*{}^2 L^2 \frac{\varepsilon''}{\Delta''^2}\left(\iint\limits_{\Sigma_1} - \iint\limits_{\Sigma_2 - S}\right)\left[\delta'' A^{-1}\left(1 + \frac{\varkappa - 1}{N_{\text{Pr}}''}\right) - \frac{1}{N_{\text{Re}}''}\right]\frac{\partial v_{x1}''}{\partial x''}\,dy''\,dz''$$

Taking into account in this formula the relationship between the small parameters δ'' and $1/N_{\text{Re}}''$, we conclude that the difference of the two terms in brackets is identically zero. The validity of equality $F_x'' = 0$ has thus been proved.

That part of the lift F_y'' which results from the integration of the external stream functions may be presented in the form

$$F_y'' = \rho_* a_*{}^2 L^2 \frac{\varepsilon''}{\Delta''}\left(\iint\limits_{\Sigma_1} - \iint\limits_{\Sigma_2 - S}\right) v_y'' dy'' dz''$$

In order to simplify computations we substitute integration over the whole plane Σ_2 for integration over area $\Sigma_2 - S$. It can be readily shown that the resulting error will be within the limits of the approximation considered. After some simple transformations based on equality

$$f(\xi) = -{}^3/_2\,\xi^{-1}[g(\xi) - h(\xi)]$$

we find

$$F_y'' = -\pi\rho_* a_*{}^2 L^2 B_1 c_2 \frac{\varepsilon''}{\Delta''}, \qquad F_z'' = -\pi\rho_* a_*{}^2 L^2 B_1 c_1 \frac{\varepsilon''}{\Delta''} \tag{3.10}$$

where constant B_1 is given by Formula (2.22).

By definition $F_i = F_i' + F_i''$. By virtue of $F_x'' = 0$ equality (3.5) yields the full magnitude of the drag acting on a body in a stream of dissipative gas having its critical velocity at infinity. The origin of this force is explained by the same reason as in the case of an incompressible fluid, and is related to the displacement of the x-component of momentum from the trail downstream of the body. Although the damping of perturbations in the external zone is sufficiently slow, their total contribution is zero. At sonic velocity of flow past a body the wave drag is absent, it occurs at purely supersonic flight velocities only [1 and 7]. The exactly opposite is true for the lift and lateral force. In fact, in accordance with (3.10) both F_y'' and F_z'' differ from zero. The generation of the lift and lateral force is, therefore, the result of the y, and z components of momentum being carried into infinity not only from the vortex trail, but also by the system of waves radiating from the body and spreading through the external stream. A part of these forces is of the wave pattern at the critical velocity of the free stream.

The relationship (2.23) between constants c_1 and c_z, and c_2 and c_y must be taken into account in the summation of the right-hand sides of Formulas (3.6) and (3.10). Expressing parameter Δ'' in terms of ε'' as defined by the first of Eqs. (3.9) we obtain

$$\begin{aligned} F_y &= -\rho_* a_*{}^2 l^2 c_y \varepsilon'\left[1 + \frac{1}{2}(2m_*)^{1/4} A^{-1/4}\left(1 + \frac{\varkappa - 1}{N_{\text{Pr}}''}\right)^{1/2}\frac{\varepsilon''^{5/4}}{\varepsilon'}\right] \\ F_z &= -\rho_* a_*{}^2 l^2 c_z \varepsilon'\left[1 + \frac{1}{2}(2m_*)^{1/4} A^{-1/4}\left(1 + \frac{\varkappa - 1}{N_{\text{Pr}}''}\right)^{1/2}\frac{\varepsilon^{5/4}}{\varepsilon'}\right] \end{aligned} \tag{3.11}$$

There remains to determine the interdependence of parameters ε' and ε'' which characterize the magnitude of perturbations in-, and outside of the accompanying trail. The simplest way of achieving this is to resort to the continuity equation in its integral form (3.3). We fix radius R of the cylindrical surface Σ_0 at a constant value, and move plane Σ_1 upstream into infinity. We take into account that in accordance with (1.21) a finite mass of gas defined by constant c_ρ may be displaced from the trail. Finally, we obtain

$$c_x + c_\rho + 2\pi B_2 \left(\frac{L}{l}\right)^2 \frac{\varepsilon''}{\varepsilon'} = 0, \qquad B_2 = \int_{-\infty}^{+\infty} g_1(\xi)\, d\xi \tag{3.12}$$

with function [3]

$$g_1 = 2\, 2^{1/3} b_3 \left[\eta^{1/3} \Phi\left(\frac{5}{3}, \frac{4}{3}; \eta\right) - \frac{\Gamma^2(1/3)}{6\Gamma^2(2/3)} \Phi\left(\frac{4}{3}, \frac{2}{3}; \eta\right)\right]$$

In our computations coefficient $b_3 = -2 \cdot 2^{1/3}\Gamma(2/3)[3\Gamma(1/3)]^{-1}$ was assumed, and the value of $B_2 = 2.002$ was obtained.

It will be seen from the first of Formulas (3.12) that the ratio of small parameters $\varepsilon''/\varepsilon'$ is of the order of unity. This formula indicates moreover that the magnitude of perturbations in the external stream may vary due to heat exchange between the body and ambient gas. Because of molecular heat transfer from the body to the boundary layer a finite mass of the medium is displaced from the latter when $c_\rho < 0$. This process is in no way related to energy dissipation on account of viscous stresses. On the contrary, when $c_\rho > 0$, a certain amount of gas is "drawn in" from the external stream into the trail downstream of the body. Constant c_x is always negative because velocity v_x' in the vortex trail must be negative In the approximation here considered the effect of drag does not, in general, produce external flow perturbations when $c_\rho = -c_x$. Such phenomenon is not possible in an incompressible fluid, as there the 'deficiency' in the trail cross-section is proportional to the magnitude of the x-component of impulse displaced from that area [1]. For a compressible medium the above phenomenon is of a general character, and is present at any flight velocity. An appreciable effect will, however, be obtained only in the presence of a sufficiently intensive heat exchange in the boundary layer.

The derivation of results presented here could have been shortened, if in all computations radius R of the cylindrical surface Σ_0 was held constant, and plane Σ_1 moved into infinity, as was done in the analysis of the continuity Eq. (3.3). Such a method has the advantage of avoiding the necessity of resorting to the second approximation theory. However in all experimental investigations involving trail traversing which were originated by Betz [12], two control surfaces $x = \text{const}$ are used. It was, therefore, thought expedient to show explicitly that in spite of the relatively considerable magnitude of perturbations in the external stream, their effect on the drag of a body is nil, even at the critical velocity of particles at infinity.

In conclusion we shall compare the results obtained here with those established in paper [3]. When a flow is axisymmetric, or a body is acted upon by drag only, then the difference $M - 1$ (M is the Mach number) decreases proportionally to $r^{-4/3}$, and the angle between the velocity vector and the x-axis proportionally to $r^{-5/3}$, when receding into infinity along line $\xi = \text{const}$. The presence of lift and lateral forces generates additional perturbations for which, under similar conditions we have in the external stream $|M - 1| \sim r^{-5/3}$ and $\omega \sim r^{-2}$. Thus the drag of a body gives rise to considerably greater perturbations in a uniform flow of gas than the perturbations generated by forces acting in the transverse plane, when motions at critical velocities are considered.

BIBLIOGRAPHY

1. Landau, L.D. and Lifshits, E.M., Mechanics of Continuous Media. 2nd Ed., M., Gostekhizdat, 1954.

2. Finn, R., On the Stokes Paradox and Related Questions. Non-linear Problems. Ed. by R.E. Langer, Madison Univ. Press Wisconsin, 1963.
3. Ryzhov, O.S., Asymptotic pattern of flow past bodies of revolution in a sonic stream of viscous and heat conducting gas, PMM, Vol. 29, No. 6, 1965.
4. Landau, L.D. and Lifshits, E.M., Statistical Physics. 2nd Ed., M., "Nauka", 1964.
5. Tollmien, W., Grenzschnichttheorie. Handbuch der Experimentalphysik. Bd. 4, Teil 1, Akadmie Verlag, Leipzig, 1931.
6. Taylor, G.I., The conditions necessary for discontinuous motions in gases. Proc. Roy. Soc., sep. A, Vol. 84, No. 571, 1910.
7. Heaslet, M.A. and Lomax, G., Theory of Small Perturbations at Supersonic and Transonic Stream Velocities. Symposium on General Theory of High Speed Aerodynamics Ed. by W.R. Sears. (Russian transl., M., Voenizdat, 1962).
8. Carslow, G.S., Theory of heat conductivity. (Russian transl. M. -L., Gostekhizdat, 1947).
9. Holder, D.W., McPhail, C.D. and Thompson, J.S., Methods of Experimental Analysis. Symposium on Present State of High Speed Aerodynamics. Ed. by L. Howard. (Russian transl. Vol. 2, Izd. Inostr. Lit., 1956).
10. Erdelyi, A., Magnus, W., Oberhettinger, F. and Tricomi, F.G., Higher Transcendental Functions, Vol. 1, McGraw-Hill, N.Y., Toronto, L., 1953.
11. Diesperov, V.N. and Ryzhov, O.S., The second approximation in the asymptotic theory of sonic flow of a real gas past bodies of revolution. PMM, Vol. 31, No. 5, 1967.
12. Betz, A., Ein Verfahren zur direkten Ermittlung des Profilwinderstandes. Z. Flugtechnik und Motorluftschiffahrt, Jg, Bd. 16, Hft. 3, 1925

Translated by J.J.D.